

Experimental and numerical study of the low temperature behaviour of LiYbF_4

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Outline

Introduction

The LiREF₄ system

Crystalline and ordered structures

Hamiltonian

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Introduction: LiREF₄

- ▶ RE = Rare Earth ions
 - ▶ magnetic : Yb, Ho, Er, Gd, Tb
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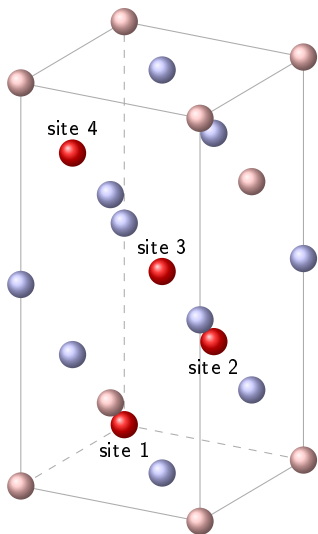
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- ▶ Possible apparition of both ferromagnetic (LiHoF₄) and antiferromagnetic (LiErF₄, LiYbF₄) order.

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- ▶ Dipolar coupled quantum magnets.
- ▶ Possible apparition of both ferromagnetic (LiHoF₄) and antiferromagnetic (LiErF₄, LiYbF₄) order.
- ▶ Disordered systems with apparition of a spin glass phase:
 - ▶ Dilution of the magnetic moments with non magnetic Y ions
 - ▶ Mixing of several RE ions with different anisotropies (LiHo_{1-x}Er_xF₄ or LiHo_{1-x}Yb_xF₄ for instance)

Crystalline structure of LiREF_4



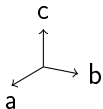
Sites coordinates


site 1: $(0, 0, 0)$


site 2: $(0, \frac{1}{2}, \frac{1}{4})$

site 3: $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$

site 4 : $(\frac{1}{2}, 0, \frac{3}{4})$



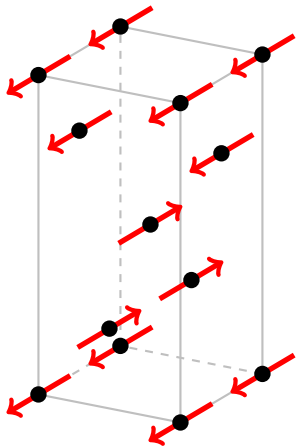
 Rare earth ions on the 4 sites in the unit cell

 Rare earth ions (in the adjacent cells)

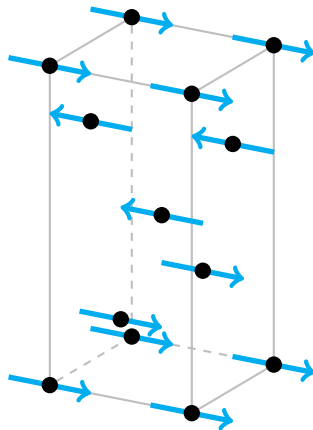
 Li ions

The Bi-Layered AntiFerroMagnetic (BLAFM) ordered structure

Moments parallel to the a-axis



Moments parallel to the b-axis



Hamiltonian of the system

$$\mathcal{H} = \mathcal{H}_{single\ ion} + \mathcal{H}_{interactions}$$

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$$\mathcal{H} = \underbrace{\mathcal{H}_{\text{single ion}}}_{\text{Crystal field}} + \mathcal{H}_{\text{interactions}}$$

$$\mathcal{H}_{cf} = \sum_i \sum_{lm} B_l^m \hat{O}_l^m(J_i)$$

Crystal field

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Crystal field

$$\mathcal{H}_Z = -\mu_B g_L \vec{J} \cdot \vec{B}$$

Zeeman term

Hamiltonian of the system

$$\mathcal{H} = \underbrace{\mathcal{H}_{\text{single ion}}}_{\substack{\mathcal{H}_{cf} = \sum_i \sum_{lm} B_l^m \hat{O}_l^m(J_i) \\ \text{Crystal field} \\ \mathcal{H}_{hyp} = A \vec{J} \cdot \vec{I} \\ \text{Hyperfine coupling} \\ \mathcal{H}_Z = -\mu_B g_L \vec{J} \cdot \vec{B} \\ \text{Zeeman term}}} + \mathcal{H}_{\text{interactions}}$$

Hamiltonian of the system

$$\mathcal{H} = \underbrace{\mathcal{H}_{\text{single ion}}}_{\text{Crystal field, Hyperfine coupling, Zeeman term}} + \underbrace{\mathcal{H}_{\text{interactions}}}_{\text{Dipolar interactions}}$$

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Crystal field

$\mathcal{H}_{hyp} = A \vec{J} \cdot \vec{I}$
Hyperfine coupling

$\mathcal{H}_Z = -\mu_B g_L \vec{J} \cdot \vec{B}$
Zeeman term

$\mathcal{H}_{dip} = -\frac{1}{2} \sum_{ij} g_{L_i} g_{L_j} \mu_B^2 \vec{J}_i D_{ij} \vec{J}_j$
Dipolar interactions

Hamiltonian of the system

$$\mathcal{H} = \underbrace{\mathcal{H}_{\text{single ion}}}_{\text{Crystal field, Zeeman term, Hyperfine coupling}} + \underbrace{\mathcal{H}_{\text{interactions}}}_{\text{Exchange interactions, Dipolar interactions}}$$

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Hyperfine coupling

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Zeeman term

$\mathcal{H}_{ex} = \sum_{i,j} \mathcal{J}_{ij} \vec{J}_i \cdot \vec{J}_j$
Exchange interactions

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Crystal field Hyperfine coupling

$\mathcal{H}_Z = -\mu_B g_L \vec{J} \cdot \vec{B}$
Zeeman term

~~$\mathcal{H}_{ex} = \sum_{i,j} J_{ij} \vec{J}_i \cdot \vec{J}_j$~~
~~Exchange interactions~~
Negligible !

$\mathcal{H}_{dip} = -\frac{1}{2} \sum_{i,j} g_{L_i} g_{L_j} \mu_B^2 \vec{J}_i D_{ij} \vec{J}_j$
Dipolar interactions

The crystal field

- ▶ Electric field created by the charge distribution around the ion
- ▶ Responsible for the magnetic anisotropy

$$\mathcal{H}_{cf} = \sum_i \sum_{lm} B_l^m \hat{O}_l^m(J_i)$$

	LiYbF ₄	LiErF ₄	LiHoF ₄
$10^3 B_2^0$	663 ± 80	60.23	-60.0
$10^3 B_4^0$	12.5 ± 4.5	-0.12	0.350
$10^3 B_4^4(c)$	102 ± 41	-4.33	3.60
$10^5 B_6^0$	-62 ± 73	-0.19	4.0
$10^3 B_6^4(c)$	-16.0 ± 1.7	-0.085	0.070
$10^6 B_6^4(s)$	0	-22.7	9.8

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	LiYbF ₄	LiYbF ₄ (new)	LiErF ₄	LiHoF ₄
$10^3 B_2^0$	663 ± 80	646.2	60.23	-60.0
$10^3 B_4^0$	12.5 ± 4.5	15.3	-0.12	0.350
$10^3 B_4^4(c)$	102 ± 41	116.5	-4.33	3.60
$10^5 B_6^0$	-62 ± 73	-68.6	-0.19	4.0
$10^3 B_6^4(c)$	-16.0 ± 1.7	-15.2	-0.085	0.070
$10^6 B_6^4(s)$	0	0	-22.7	9.8

Mean field approximation

- ▶ Fluctuations are neglected and effective mean fields are introduced :

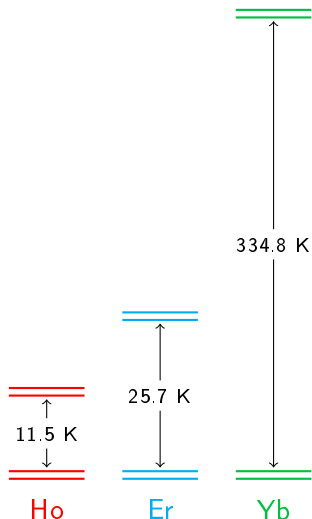
$$\vec{h}_{\text{eff}}^i = \sum_{j=1}^4 \tilde{D}_{ij} \langle \mu_B \mathbf{g}_{L_j} \cdot \vec{J}_j \rangle$$

- ▶ Hamiltonians decoupled for each site :

$$\mathcal{H}_{\text{dip}}^{\text{MF}} = \sum_{i=1}^4 \mathbf{g}_{L_i} \mu_B \vec{J}_i \cdot \vec{h}_{\text{eff}}^i$$

- ▶ Algorithm :
 - ▶ Computation of \tilde{D}_{ij}
 - ▶ Initialization of the moments $\langle J_i \rangle$
 - ▶ Computation of h_{eff}^i for the current configuration
 - ▶ Diagonalization of $\mathcal{H}_i^{\text{MF}}$ and update of $\langle J_i \rangle$
 - ▶ Evaluation of $\Delta = \sum_i |\langle J_i \rangle^{\text{new}} - \langle J_i \rangle^{\text{old}}|$

Monte Carlo calculation: the quantum effective model



Effective model for $\mathcal{H}_0 = \mathcal{H}_{cf} + \mathcal{H}_Z$

- ▶ Diagonalization of \mathcal{H}_0
- ▶ Projection of J and \mathcal{H}_0 on the subspace of the doublet
- ▶ Diagonalization of \tilde{J}_z
- ▶ Obtention of a basis $\{|+\rangle, |-\rangle\}$

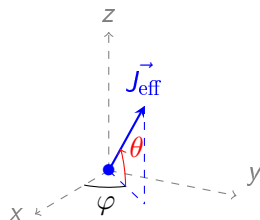
State:

$$|\alpha, \beta\rangle = \cos(\alpha) |+\rangle + e^{i\beta} \sin(\alpha) |-\rangle$$

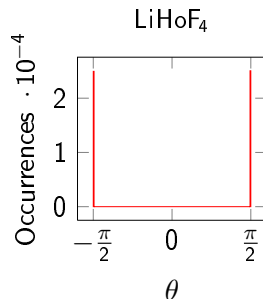
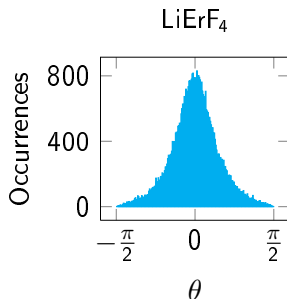
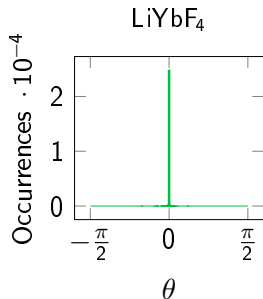
where $\alpha \in [0, \frac{\pi}{2}]$ and $\beta \in [0, 2\pi]$.

The classical effective model

- ▶ 2 parameters: α and β
- ▶ Classical moment: $\vec{J}_{eff} = \langle \alpha, \beta | \vec{J} | \alpha, \beta \rangle$

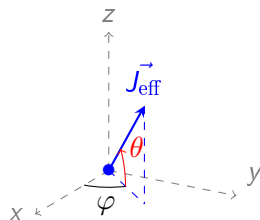


Distribution of θ



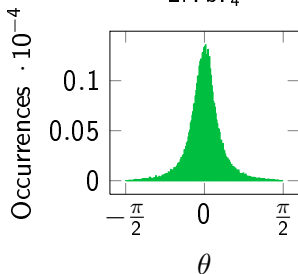
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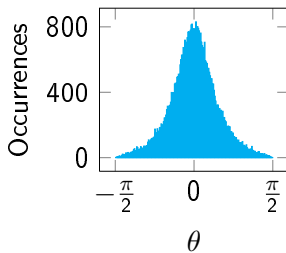


Distribution of θ

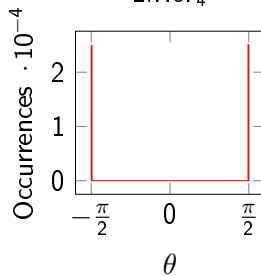
LiYbF₄



LiErF₄



LiHoF₄



Critical exponents

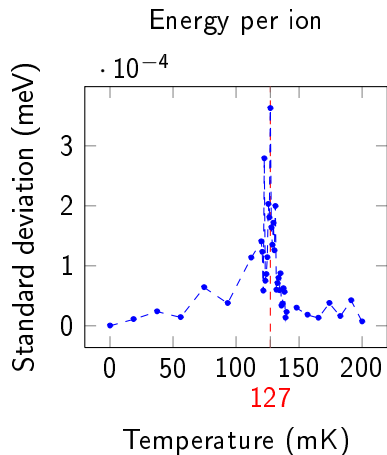
$$C \propto |T - T_N|^{-\alpha}$$

$$J_{xy}^{alt} \propto |T - T_N|^{\beta}$$

$$\chi \propto |T - T_N|^{-\gamma}$$

Exponent		α	β	γ
Mean-field		0	0.5	1
3D	Ising	-0.11	0.32	1.24
	XY	0.01	0.35	1.32
	Heisenberg	0.12	0.36	1.39
2D	Ising	0	0.125	1.75
	XY/h4		0.1-0.25	
	LiErF ₄ (at T_N)	0.28 ± 0.04	0.15 ± 0.02	0.82 ± 0.04
	LiErF ₄ (at H_c)		0.31 ± 0.02	1.44 ± 0.2

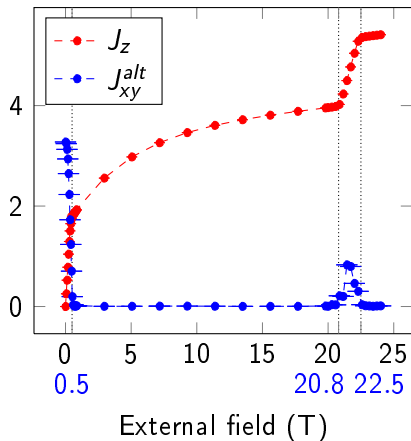
Monte Carlo simulation of LiErF_4



● $T_N = 127$ mK Exp: 373 mK

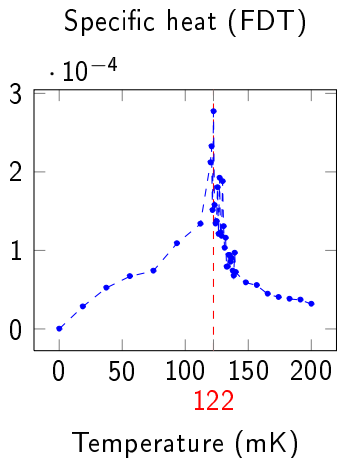
Monte Carlo simulation of LiErF_4

Quantum phase transition



- $T_N = 127$ mK Exp: 373 mK
- $H_C = 0.5$ T Exp: 0.4 T

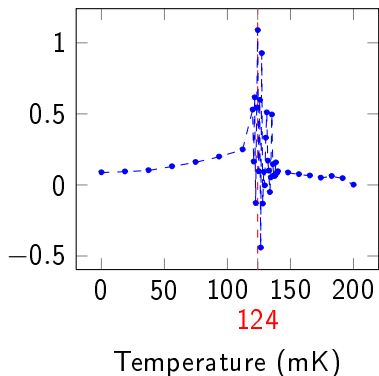
Monte Carlo simulation of LiErF_4



- T_N = 127 mK Exp: 373 mK
- H_C = 0.5 T Exp: 0.4 T
- α_{FDT} = 0.33 Exp: 0.28

Monte Carlo simulation of LiErF_4

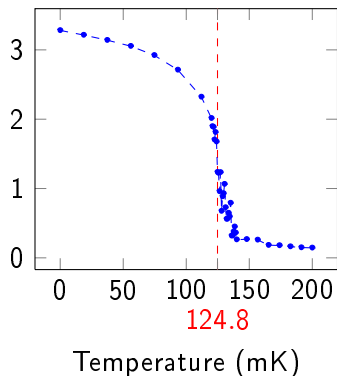
Specific heat



●	T_N	= 127 mK	Exp: 373 mK
●	H_C	= 0.5 T	Exp: 0.4 T
●	α_{FDT}	= 0.33	Exp: 0.28
●	α	= 0.27	Exp: 0.28

Monte Carlo simulation of LiErF_4

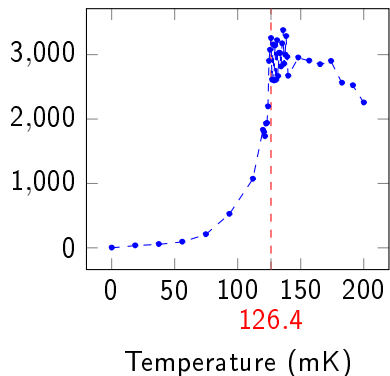
Order parameter



●	T_N	= 127 mK	Exp: 373 mK
●	H_C	= 0.5 T	Exp: 0.4 T
●	α_{FDT}	= 0.33	Exp: 0.28
●	α	= 0.27	Exp: 0.28
●	β	= 0.15	Exp: 0.15

Monte Carlo simulation of LiErF_4

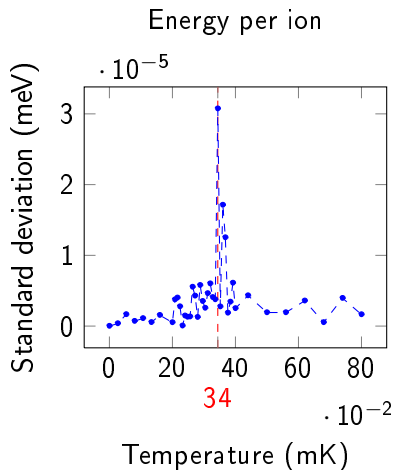
Susceptibility



●	T_N	= 127 mK	Exp: 373 mK
●	H_C	= 0.5 T	Exp: 0.4 T
●	α_{FDT}	= 0.33	Exp: 0.28
●	α	= 0.27	Exp: 0.28
●	β	= 0.15	Exp: 0.15
●	γ	= 0.91	Exp: 0.82

Monte Carlo simulation of LiYbF₄

Wrong CF parameters !

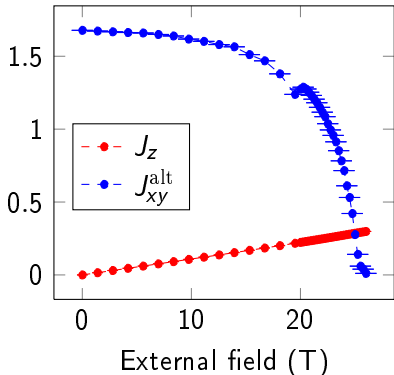


● $T_N = 34.4$ mK Exp: 128.6 mK

Monte Carlo simulation of LiYbF_4

Wrong CF parameters !

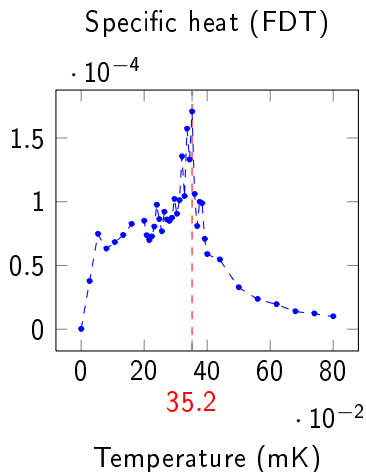
Quantum phase transition



- $T_N = 34.4$ mK Exp: 128.6 mK
- $H_C = 25$ T Exp: 0.48 T

Monte Carlo simulation of LiYbF₄

Wrong CF parameters !

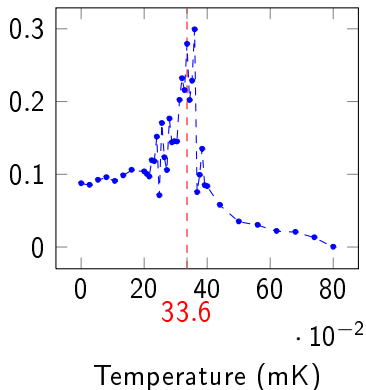


- T_N = 34.4 mK Exp: 128.6 mK
- H_C = 25 T Exp: 0.48 T
- α_{FDT} = 0.25

Monte Carlo simulation of LiYbF₄

Wrong CF parameters !

Specific heat

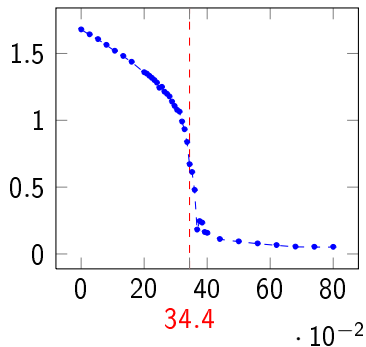


- T_N = 34.4 mK Exp: 128.6 mK
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Monte Carlo simulation of LiYbF_4

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Order parameter

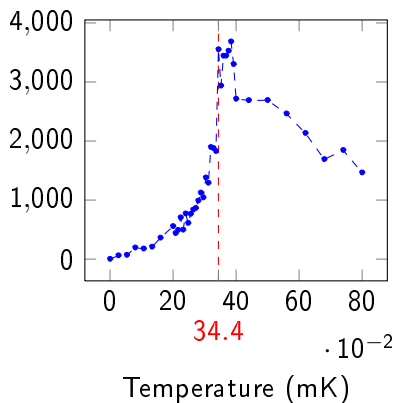


●	T_N	= 34.4 mK	Exp: 128.6 mK
●	H_C	= 25 T	Exp: 0.48 T
●	α_{FDT}	= 0.25	
●	α	= 0.28	
●	β	= 0.18	

Monte Carlo simulation of LiYbF_4

Wrong CF parameters !

Susceptibility

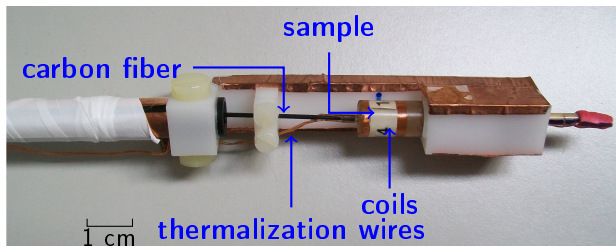
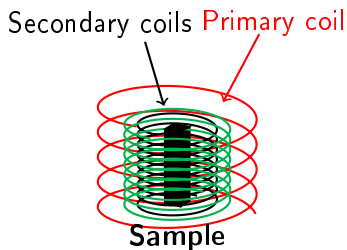


●	T_N	= 34.4 mK	Exp: 128.6 mK
●	H_C	= 25 T	Exp: 0.48 T
●	α_{FDT}	= 0.25	
●	α	= 0.28	
●	β	= 0.18	
●	γ	= 0.90	

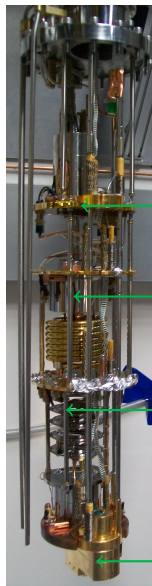
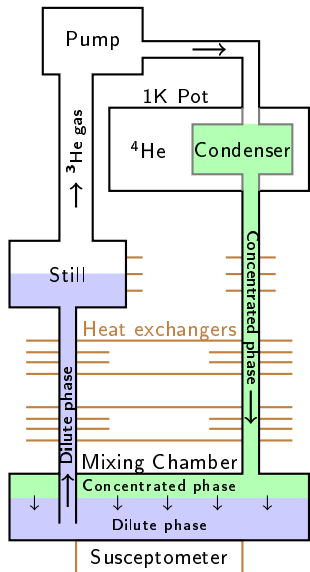
AC susceptibility measurements

$$\chi' = \frac{\omega}{\mu_0 H_{a0} \pi} \int_0^{\frac{2\pi}{\omega}} \langle B \rangle \cos(\omega t) dt - 1$$

$$\chi'' = \frac{\omega}{\mu_0 H_{a0} \pi} \int_0^{\frac{2\pi}{\omega}} \langle B \rangle \sin(\omega t) dt$$

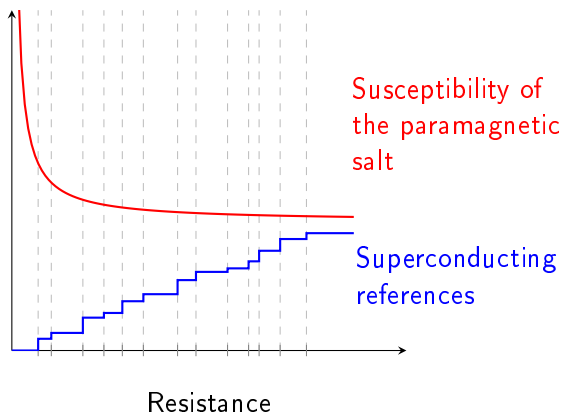


The dilution refrigerator

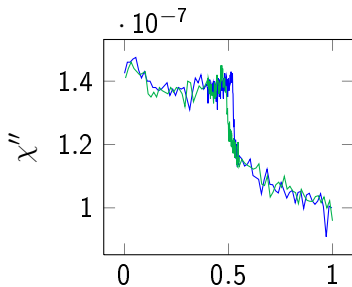
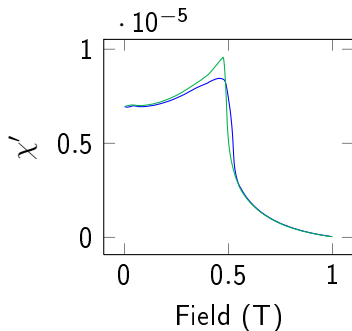
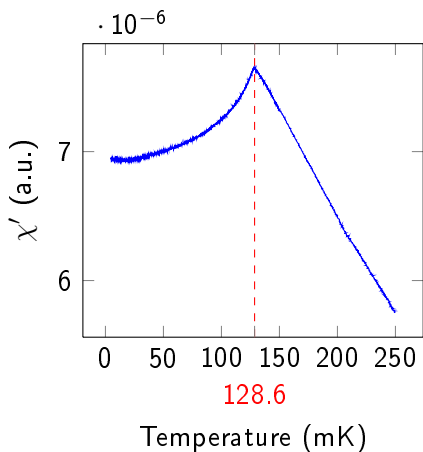


Measurement of the temperature

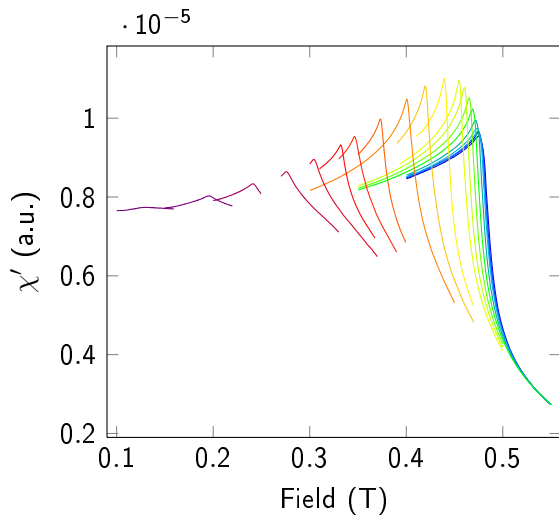
- ▶ Thermometer: thick film of RuO_x
- ▶ Calibration with 13 superconducting references and a paramagnetic salt.



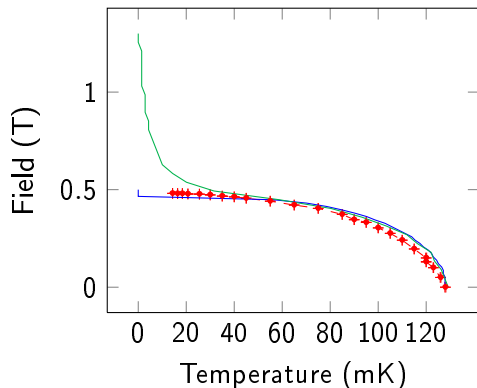
Critical field and temperature



Field scans



Phase diagram



- *-- Experiment
- MF without hyperfine coupling (T rescaled)
- MF with hyperfine coupling (T rescaled)

	T_N (mK)	exp.
Experiment	128	0.32
MF no hyp	186	0.24
MF hyp	182	

	H_C (T)	exp.
Experiment	0.48	0.54
MF no hyp	0.46	0.52
MF hyp	1.2	0.5

Future work

- ▶ Monte Carlo simulation for LiYbF_4 with the new parameters
- ▶ Study of the disordered compounds $\text{LiHo}_{1-x}\text{Yb}_x\text{F}_4$
 - ▶ Monte Carlo simulation
 - ▶ Measurement of the T_C - x phase diagram